Leader-Following Consensus of Nonlinear Multiagent Systems With Stochastic Sampling

Wangli He, *Member, IEEE*, Biao Zhang, Qing-Long Han, *Senior Member, IEEE*, Feng Qian, Jürgen Kurths, and Jinde Cao, *Fellow, IEEE*

***Abstract*—This paper is concerned with sampled-data leader-following consensus of a group of agents with nonlinear characteristic. A distributed consensus protocol with probabilistic sampling in two sampling periods is proposed. First, a general consensus criterion is derived for multiagent systems under a directed graph. A number of results in several special cases without transmittal delays or with the deterministic sampling are obtained. Second, a dimension-reduced condition is obtained for multiagent systems under an undirected graph. It is shown that the leader-following consensus problem with stochastic sampling can be transferred into a master–slave synchronization problem with only one master system and two slave systems. The problem solving is independent of the number of agents, which greatly facilitates its application to large-scale networked agents. Third, the network design issue is further addressed, demonstrating the positive and active roles of the network structure in reaching consensus. Finally, two examples are given to verify the theoretical results.**

***Index Terms*—Leader-following consensus, multiagent system, sampled-data control, stochastic sampling.**

I. Introduction

In recent years, consensus of multiagent systems has received considerable attention due to its wide applications in cooperative control of unmanned air vehicles, formation control in robotic systems, and distributed sensing in sensor networks [1]–[3]. It is well known that consensus can be reached via sufficient local information exchanges between agents and their neighbors.

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W. He, B. Zhang, and F. Qian are with the Key Laboratory of Advanced Control and Optimization for Chemical Processes, Ministry of Education, East China University of Science and Technology, Shanghai 200237, China (e-mail: wanglihe@ecust.edu.cn; fqian@ecust.edu.cn).

Q.-L. Han is with the Griffith School of Engineering, Griffith University, Gold Coast, QLD 4222, Australia (e-mail: q.han@griffith.edu.au).

J. Kurths is with the Potsdam Institute for Climate Impact Research, Telegraphenberg, Potsdam D-14415, Germany, and also with the Institute for Complex Systems and Mathematical Biology, University of Aberdeen, Aberdeen AB24 3UE, U.K. (e-mail: kurths@pik-potsdam.de).

J. Cao is with the Department of Mathematics, Southeast University, Nanjing 210096, China (e-mail: jdcao@seu.edu.cn).

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There have been extensive studies on distributed control protocols for achieving consensus among a group of autonomous agents [4]–[9]. Most of these studies utilize continuous measurement signals for consensus protocol design [4]–[9]. However, in most real-world applications, continuous communication is expensive, or even unavailable since there are still some technical challenges in keeping reliable continuous communication network [10]. On the other hand, communication channels are usually occupied in continuous communication, the limited bandwidth of the communication network, and power source cannot afford the communication burden. An energy-saving consensus protocol involving discrete measurement signals is in great demand [11], [12]. Sampled-data control is regarded as one of effective control strategies with only discrete signals required [13]–[15]. With sampled position and/or velocity data, consensus was investigated for multiagent systems with double-integrator dynamic [16]–[21]. In [16]–[18], some necessary and sufficient conditions for second-order consensus were established in which exact algebraic expressions involving the sampling interval and the eigenvalues of the Laplacian matrix associated with the network topology were given. The relationship between the sampling interval and the network topology in determining consensus was clearly shown. Note that the results in [16]–[18] only hold for multiagent systems with second-order linear dynamics. However, in practice, intelligent agents are more likely to be governed by complicated intrinsic nonlinear dynamics. Indeed, nonlinear dynamics are widely studied in synchronization of complex networks [22]–[24]. In [25], nonlinear multiagent systems were employed into the problem of sampled-data leader-following consensus with random switching network topologies and communication delays. The work in [26] discussed H_∞ pinning synchronization of leader-following nonlinear networks. The leaderless case was also investigated in [27] with a fixed network topology. In [28], distributed impulsive control was applied to leader-following heterogenous dynamic networks. It should be pointed out the sampling scheme in the aforementioned work belongs to the deterministic sampling, in which the sampling interval is periodic or aperiodic with an upper bound.

As the large sampling interval means signals are sampled during a relative long time period, less energy is consumed. Thus, such a sampling scheme with less signals sampled is more efficient. Stochastic sampling, which allows the sampling period to switch among different values, has received increasing attention. It was shown that larger sampling intervals were derived in stabilizing a linear system [29], compared with the deterministic sampling. The extension to synchronization of coupled nonlinear systems was reported in [30]. Recently, stochastic sampling has been applied to networked Euler–Lagrange systems [31] and networked linear multiagent systems [32]. It is worth pointing out that results for networked nonlinear systems are limited to Euler–Lagrange systems [31] or leader-following consensus with only sampled leader's signal. Continuous communication between follower agents was still required in [30]. Moreover, the transmittal delays between agents are neglected [16]–[21], [29]–[32]. Therefore, developing a practical consensus protocol with stochastic sampling for networked nonlinear systems and further investigating the effects of transmittal delays on consensus are one motivation of this paper.

This paper aims at studying leader-following consensus of networked nonlinear multiagent systems with stochastic sampling. The transmittal delays between agents are considered. The fundamental problem on how to design the stochastic sampling scheme involving two or more sampling periods and their selection probabilities for consensus of a group of nonlinear multiagent system is studied. It should be pointed out that for a single linear system, the design of the stochastic sampling scheme is not easy [29]. The sampling parameters are often involved in a complex matrix inequality and are not easy to be identified. It becomes more challenging when networked systems are considered due to the complicated interactions represented by a network topology matrix. In such cases, sampling parameters are not only related to the system dynamics, but also the network topology. High-dimensional matrix inequalities are derived to determine sampling parameters [25], [30], [32]. As the dimension of derived matrix inequalities depends on the number of agents [25]–[27], [30], [32], the problem solving is not available when large-scale multiagent systems are involved. Fortunately, some matrix decomposition and variable substitution techniques have been proposed for linear multiagent systems with determined sampling [16]–[18], resulting in low-dimensional algebraic criteria, explicitly showing the relationship between the sampling period and the eigenvalues of the Laplacian matrix. As these techniques cannot be directly extended to nonlinear systems, high-dimensional criteria are still used in [25]–[27], [30], and [32], which led to the investigation of nonlinear multiagent systems aiming at deriving low-dimensional conditions and further exploring the relationship between the sampling scheme and the network parameters. On the other hand, the effect of network parameters is not well explored in the aforementioned literature. They mainly focus on how to design the sampling scheme with a given network while how to design the network corresponding to the sampling scheme is seldom mentioned, which is the other motivation of this paper.

In this paper, we considers leader-following consensus of nonlinear multiagent systems with stochastic sampling under a directed graph. An efficient distributed consensus protocol with stochastic sampling and transmittal delays is proposed. The sampling data of agents and their neighbors at only discrete-time instants is used and the pinning technique is also involved for saving energy and lowering communication cost. A general criterion is first derived to ensure leader-following consensus in the mean-square sense. The effects of transmittal delays and comparisons with the deterministic sampling are discussed within several corollaries. Then, a low-dimensional criterion for multiagent systems with an undirected graph is obtained. Furthermore, the network design problem is also addressed, demonstrating the positive and active roles of the network structure in reaching consensus. Finally, two examples are given to verify the theoretical results.

A. Notations

$\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix two-norm. For symmetric matrices P and Q, the notation $P < Q$ means that the matrix $P - Q$ is negative definite. $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represents the maximum (minimum) eigenvalue of the symmetric matrix A. I_n is the identity matrix of order n. $\mathcal{C}([a, b], \mathbb{R}^n]$ denotes the Banach space of continuous vector-valued functions mapping the interval $[a, b]$ into $\mathbb{R}^n$. "$*$" denotes the entries implied by the symmetry of a matrix.

II. Preliminaries and Problem Statement

In this section, some basic concepts about algebraic graph theory are first introduced.

Let $G = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ be a directed weighted graph consisting of a set of vertices $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, a set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{W} = (w_{ij})$ with nonnegative entries. An edge e_{ij} is denoted by an ordered pair of vertices (v_j, v_i), where v_j and v_i are called the parent and child vertices, respectively, and $e_{ij} \in \mathcal{E}$ if and only if $w_{ij} > 0$. A path from node v_i to node v_j is a sequence of edges (v_j, v_{i_1}), $(v_{i_1}, v_{i_2}), \dots, (v_{i_p}, v_i)$, with distinct nodes i_k, $k = 1, 2, \dots, p$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ associated with an adjacency matrix $\mathcal{W}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} w_{ij}$ and $l_{ij} = -w_{ij}$ for $i \neq j$.

A. Leader-Following Multiagent System

Consider a leader-following multiagent system with one leader and N followers. The leader is described by

$$\dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)) \tag{1}$$

where $x_0(t) \in \mathbb{R}^n$ denotes the state of the leader; $f(x_0(t)) = (f_1(x_0(t)),\ f_2(x_0(t)), \dots, f_n(x_0(t)))^{\mathrm{T}}$ is a nonlinear function, and A and B are constant matrices.

The dynamics of the follower agents are governed by

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + u_i(t),\ \ i = 1, 2, \dots, N \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of the ith agent and $u_i(t)$ is the controller to be designed.

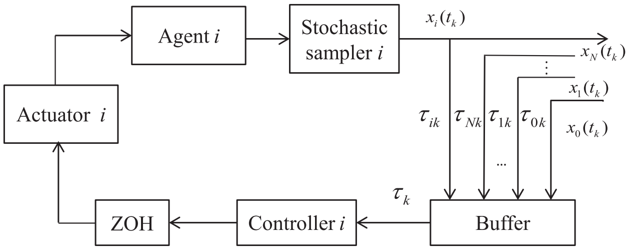

Fig. 1. Stochastic sampling framework for the multiagent system.

Assumption 1: The nonlinear function $f(\cdot)$ satisfies the Lipschitz condition, i.e., there exist non-negative constants h_{ij} $(i, j = 1, 2, \ldots, n)$ such that for any $z_1, z_2 \in \mathbb{R}^n$

$$|f_i(z_1) - f_i(z_2)| \leq \sum_{j=1}^{n} h_{ij} \left| z_{1j} - z_{2j} \right|.$$

Assumption 2: There exists a path from the leader to every follower agent.

B. Consensus Protocol With Stochastic Sampling

In this paper, an efficient consensus protocol is introduced with sampled data. We assume that the sampling period is allowed to randomly switch between two different values. Such a sampling scheme is referred as stochastic sampling. A basic stochastic sampling framework for the multiagent system is illustrated in Fig. 1.

At the sampling instant t_k, the state of agent i is sampled and the next sampling instant t_{k+1} is generated according to occurrence probabilities of these two sampling periods, which is implemented by stochastic sampler. It is assumed that all the agents are sampled synchronously. The transmittal delay from the sensor to the controller is considered. Note that packet dropouts and disorders may happen in the process of information transmission. However, in order to simplify the analysis, only the effect of transmittal delays is considered in this paper. As transmittal delay varies from agent to agent, a common buffer is employed to make all controllers operate at the same time. Let $\tau_{ik}, \ i = 0, 1, 2, \ldots, N$ be the communication delay between the sensor i and the buffer. Thus, the delay from the sensor i to the controller i can be defined as $\tau_k = \max\{\tau_{ik}, \ i = 0, 1, 2, \ldots, N\}$. The controller i updates its input and sends its output to the actuator with zero-order hold (ZOH). The function of ZOH is to keep the control input constant from $t = t_k + \tau_k$ to $t = t_{k+1} + \tau_{k+1}$. Then, a sample-based consensus protocol is designed as

$$u_i(t) = -c \sum_{j=1}^{N} l_{ij} x_j(t_k) - c k_i (x_i(t_k) - x_0(t_k)) \tag{3}$$

for $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, where $0 = t_0 < t_1 < \cdots < t_k < \ldots$, $\lim_{k\to\infty} t_k = \infty$, c is the coupling strength, and $k_i \geq 0, \ i = 1, 2, \ldots, N$, are pinning gains. Here, $k_i > 0$ if and only if the ith node is informed by the leader. The node i is referred to as the pinned node or controlled node.

Remark 1: The controller (3) implies all the agents share the same sampling sequence $\{t_0, t_1, t_2, \ldots\}$. If a periodic sampling is employed, time synchronization scheme can be applied to adjust sensors with synchronous sampling instant, which has been studied in [3] and [34]. For a stochastic sampling in this paper, asynchronous sampling may be more practical [11]. To relax the condition of synchronous sampling, it is expected some event-trigged scheme is introduced to ensure every agent has its own update time, which will be considered in a near future study.

Let $d(t) = t - t_k$ for $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$. As $t_k = t - (t - t_k) = t - d(t)$, then the controller (3) can then be written as

$$u_i(t) = -c \sum_{j=1}^{N} l_{ij} x_j(t - d(t)) - c k_i (x_i(t - d(t)) - x_0(t - d(t))) \tag{4}$$

with $\tau_k \leq d(t) < t_{k+1} - t_k + \tau_{k+1}$.

The sampling period $\{t_{k+1} - t_k\}$ is allowed to randomly switch between two different values p_1 and p_2 with $0 < p_1 < p_2$. The probabilities are $\text{Prob}\{p = p_1\} = \beta$ and $\text{Prob}\{p = p_2\} = 1 - \beta$, where $\beta \in [0, 1]$ is a given constant. Therefore, the time delay $d(t)$ in (4) satisfies

$$\tau_k \leq d(t) < p_1 + \tau_{k+1}$$

or

$$\tau_k \leq d(t) < p_2 + \tau_{k+1}.$$

Since the sampling period can switch between p_1 and p_2, $d(t)$ is a random variable ranging from τ_k to $p_2 + \tau_{k+1}$. The probability of $d(t)$ can be calculated by

$$\text{Prob}\{\tau_k \leq d(t) < p_1 + \tau_{k+1}\} = \beta + \left(1 - \frac{p_2 - p_1}{p_2 + \tau_{k+1} - \tau_k}\right)(1 - \beta) \tag{5}$$

$$\text{Prob}\{p_1 + \tau_{k+1} \leq d(t) < p_2 + \tau_{k+1}\} = \frac{p_2 - p_1}{p_2 + \tau_{k+1} - \tau_k}(1 - \beta). \tag{6}$$

The probability distribution in (5) and (6) depends on transmittal delays, which brings a big challenge for the stability analysis. To simplify the analysis, it is assumed that transmittal delays are uniform, that is, $\tau_k = \tau$ for $k = 0, 1, 2, \ldots$ Then (5) and (6) turn to

$$\text{Prob}\{\tau \leq d(t) < p_1 + \tau\} = \beta + (p_1/p_2)(1 - \beta)$$
$$\text{Prob}\{p_1 + \tau \leq d(t) < p_2 + \tau\} = (1 - p_1/p_2)(1 - \beta).$$

By introducing a new random variable

$$\alpha(t) = \begin{cases} 1, & \tau \leq d(t) < p_1 + \tau \\ 0, & p_1 + \tau \leq d(t) < p_2 + \tau. \end{cases}$$

Accordingly, $\text{Prob}\{\alpha(t) = 1\} = \alpha$ and $\text{Prob}\{\alpha(t) = 0\} = 1 - \alpha$ with $\alpha = \beta + (p_1/p_2)(1 - \beta)$. $\alpha(t)$ satisfies a Bernoulli distribution with $\mathbb{E}\{\alpha(t)\} = \alpha$ and $\mathbb{E}\{(\alpha(t) - \alpha)^2\} = \alpha(1 - \alpha)$.

Define the error signal $e_i(t) = x_i(t) - x_0(t)$. The controller (4) can be converted into

$$u_i(t) = -c\alpha(t)\left(\sum_{j=1}^{N} l_{ij}e_j(t-\tau_1(t)) + k_ie_i(t-\tau_1(t))\right) - c(1-\alpha(t))\left(\sum_{j=1}^{N} l_{ij}e_j(t-\tau_2(t)) + k_ie_i(t-\tau_2(t))\right) \quad (7)$$

where $\tau_1(t)$ and $\tau_2(t)$ are time-varying delays, satisfying $\tau \leq \tau_1(t) < p_1 + \tau$ and $p_1 + \tau \leq \tau_2(t) < p_2 + \tau$.

C. Error System

From (2) and (7), one has the error dynamics as

$$\dot{e}_i(t) = Ae_i(t) + Bg(e_i(t), s(t)) - c\alpha(t)\left(\sum_{j=1}^{N} l_{ij}e_j(t-\tau_1(t)) + k_ie_i(t-\tau_1(t))\right) - c(1-\alpha(t))\left(\sum_{j=1}^{N} l_{ij}e_j(t-\tau_2(t) + k_ie_i(t-\tau_2(t))\right) \quad (8)$$

where $g(e_i(t), s(t)) = f(e_i(t)+s(t)) - f(s(t))$.

Denote $e(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, $e(t-\tau_i(t)) = (e_1^{\mathrm{T}}(t-\tau_i(t)), e_2^{\mathrm{T}}(t-\tau_i(t)), \ldots, e_N^{\mathrm{T}}(t-\tau_i(t)))^{\mathrm{T}}$, $i = 1, 2$, and $g(e(t), s(t)) = (g^{\mathrm{T}}(e_1(t), s(t)), \ldots, g^{\mathrm{T}}(e_N(t)), s(t))^{\mathrm{T}}$. One has

$$\begin{aligned}\dot{e}(t) = {} & (I_N \otimes A)e(t) + (I_N \otimes B)g(e(t), s(t)) \\ & - c\alpha(t)((L+K) \otimes I_n)e(t-\tau_1(t)) \\ & - c(1-\alpha(t))((L+K) \otimes I_n)e(t-\tau_2(t))\end{aligned} \quad (9)$$

where $K = \mathrm{diag}\{k_1, k_2, \ldots, k_N\}$.

The initial condition of system (9) is defined by

$$e(\theta) = \varphi(\theta), \quad -p_2 - \tau \leq \theta \leq 0, \quad i = 1, 2, \ldots, N$$

where $\varphi(\theta) = (\varphi_1^{\mathrm{T}}(\theta), \varphi_2^{\mathrm{T}}(\theta), \ldots, \varphi_N^{\mathrm{T}}(\theta))^{\mathrm{T}}$ and $\varphi_i(\theta) \in \mathcal{C}([-p_2-\tau, 0], \mathbb{R}^n)$.

Definition 1: The multiagent system (2) is said to achieve exponential consensus with the leader $x_0(t)$ in the mean square, i.e., the error system (9) achieves exponential stability in the mean square, if there exist two positive constants μ and ε such that for any $\varphi_i(\theta) \in \mathcal{C}([-p_2-\tau, 0], \mathbb{R}^n)$

$$\mathbb{E}\left\{\|e(t)\|^2\right\} \leq \mu \sup_{-p_2-\tau\leq\theta\leq 0} \mathbb{E}\left\{\|\varphi(\theta)\|^2\right\} e^{-\varepsilon t}.$$

III. Consensus Criteria

In this section, a general sufficient sampled-data consensus condition is derived first. Then, several special cases are discussed and a low-dimensional criterion for multiagent systems with an undirected graph is obtained. Finally, the design issue of the pinning matrix and the coupling strength is addressed.

A. Consensus Under Directed Graph

Choose the Lyapunov–Krasovskii functional candidate as

$$V(t, e_t) = V_1(t, e_t) + V_2(t, e_t) + V_3(t, e_t) \quad (10)$$

where

$$\begin{aligned}V_1(t, e_t) = {} & e^{\mathrm{T}}(t)(I_N \otimes P)e(t) \\ V_2(t, e_t) = {} & \int_{t-\tau}^{t} e^{\mathrm{T}}(s)(I_N \otimes Q_1)e(s)\mathrm{d}s \\ & + \int_{t-p_1-\tau}^{t-\tau} e^{\mathrm{T}}(s)(I_N \otimes Q_2)e(s)\mathrm{d}s \\ & + \int_{t-p_2-\tau}^{t-p_1-\tau} e^{\mathrm{T}}(s)(I_N \otimes Q_3)e(s)\mathrm{d}s \\ V_3(t, e_t) = {} & \tau \int_{-\tau}^{0}\int_{t+\omega}^{t}\big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_1)\varpi(s) \\ & + \rho^{\mathrm{T}}(s)(I_N \otimes R_1)\rho(s)\big)\mathrm{d}s\,\mathrm{d}\omega \\ & + p_1 \int_{-p_1-\tau}^{-\tau}\int_{t+\omega}^{t}\big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_2)\varpi(s) \\ & + \rho^{\mathrm{T}}(s)(I_N \otimes R_2)\rho(s)\big)\mathrm{d}s\,\mathrm{d}\omega \\ & + (p_2-p_1) \int_{-p_2-\tau}^{-p_1-\tau}\int_{t+\omega}^{t}\big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_3)\varpi(s) \\ & + \rho^{\mathrm{T}}(s)(I_N \otimes R_3)\rho(s)\big)\mathrm{d}s\,\mathrm{d}\omega\end{aligned}$$

with $e_t = e(t+\theta)$, $\forall\ \theta \in [-p_2-\tau, 0]$, symmetric matrices $P > 0$, $Q_i > 0$, $R_i > 0$ $(i = 1, 2, 3)$, and

$$\begin{aligned}\varpi(t) = {} & (I_N \otimes A)e(t) + (I_N \otimes B)g(t, e(t)) \\ & - c\alpha((L+K) \otimes I_n)e(t-\tau_1(t)) \\ & - c(1-\alpha)((L+K) \otimes I_n)e(t-\tau_2(t)) \\ \rho(t) = {} & c\sqrt{\alpha(1-\alpha)}((L+K) \otimes I_n)(e(t-\tau_1(t)) \\ & - e(t-\tau_2(t))).\end{aligned}$$

We now state and establish the following theorem.

Theorem 1: Under Assumptions 1 and 2, the error system (9) is exponentially mean-square stable if there exist matrices $P > 0$, $Q_i > 0$, and $R_i > 0$ $(i = 1, 2, 3)$, a diagonal matrix $S > 0$, and constants $p_1 > 0$, $p_2 > 0$, $c > 0$, and $\beta \geq 0$ such that

$$\Omega_1 < 0 \quad (11)$$

where Ω_1 is given at the top of the next page with

$$\begin{aligned}\Omega_{11}^1 &= I_N \otimes \left(PA + A^{\mathrm{T}}P + Q_1 + HSH - R_1 + A^{\mathrm{T}}\digamma A\right) \\ \Omega_{17}^1 &= I_N \otimes \left(PB + A^{\mathrm{T}}\digamma B\right) \\ \Omega_{22}^1 &= I_N \otimes (Q_2 - Q_1 - R_1 - R_2) \\ \Omega_{33}^1 &= -2I_N \otimes R_2 + c^2\alpha(L+K)^{\mathrm{T}}(L+K) \otimes \digamma \\ \Omega_{37}^1 &= -c\alpha(L+K)^{\mathrm{T}} \otimes \digamma B \\ \Omega_{44}^1 &= I_N \otimes (Q_3 - Q_2 - R_2 - R_3) \\ \Omega_{55}^1 &= -2I_N \otimes R_3 + c^2(1-\alpha)(L+K)^{\mathrm{T}}(L+K) \otimes \digamma \\ \Omega_{57}^1 &= -c(1-\alpha)(L+K)^{\mathrm{T}} \otimes \digamma B \\ \Omega_{66}^1 &= I_N \otimes (-Q_3 - R_3) \\ \Omega_{77}^1 &= I_N \otimes \left(-S + B^{\mathrm{T}}\digamma B\right) \\ \digamma &= \tau^2R_1 + p_1^2R_2 + (p_2-p_1)^2R_3\end{aligned}$$

and $H = (h_{ij})_{n\times n}$.

$$\Omega_1 = \begin{pmatrix} \Omega_{11}^1 & I_N \otimes R_1 & -c\alpha(L+K)\otimes(P+A^{\mathrm{T}}\digamma) & 0 & -c(1-\alpha)(L+K)\otimes(P+A^{\mathrm{T}}\digamma) & 0 & \Omega_{17}^1 \\ * & \Omega_{22}^1 & I_N \otimes R_2 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33}^1 & I_N \otimes R_2 & 0 & 0 & \Omega_{37}^1 \\ * & * & * & \Omega_{44}^1 & I_N \otimes R_3 & 0 & 0 \\ * & * & * & * & \Omega_{55}^1 & I_N \otimes R_3 & \Omega_{57}^1 \\ * & * & * & * & * & \Omega_{66}^1 & 0 \\ * & * & * & * & * & 0 & \Omega_{77}^1 \end{pmatrix}$$

Proof: See Appendix. ■

Remark 2: The stochastic sampling, as a special type of sampling processes, has received more attention in the area of coupled dynamical networks in recent years. Compared with [30], an energy-saving strategy combing with the pinning technique and sampling communication between agents is proposed in this paper. Although the same scheme is presented in [26], it restricts the study on deterministic sampling. Both results presented in [26] and [30] cannot deal with the case with stochastic sampling as discussed in this paper.

Furthermore, if $\beta = 0$ or $\beta = 1$, the stochastic sampling problem is reduced to a deterministic sampling with a sampling period p. Then, the error system becomes

$$\dot{e}(t) = (I_N \otimes A)e(t) + (I_N \otimes B)g(t, e(t)) - c((L+K)\otimes I_n)e(t-\tau_1(t)) \quad (12)$$

where $\tau < \tau_1(t) \leq p + \tau$. The corresponding Lyapunov–Krasovskii functional is chosen as

$$\begin{aligned} V(t, e_t) &= e^{\mathrm{T}}(t)(I_N \otimes P)e(t) + \int_{t-\tau}^{t} e^{\mathrm{T}}(s)(I_N \otimes Q_1)e(s) \\ &+ \int_{t-p-\tau}^{t-\tau} e^{\mathrm{T}}(s)(I_N \otimes Q_2)e(s) \\ &+ \tau \int_{-\tau}^{0}\int_{t+\omega}^{t} \big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_1)\varpi(s) \\ &\qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_1)\rho(s)\big)\mathrm{d}s\,\mathrm{d}\omega \\ &+ p \int_{-p}^{0}\int_{t+\omega}^{t} \big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_2)\varpi(s) \\ &\qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_2)\rho(s)\big)\mathrm{d}s\,\mathrm{d}\omega. \end{aligned}$$

By Theorem 1, we have the following corollary.

Corollary 1: Under Assumptions 1 and 2, the error system (12) is exponentially stable if there exist matrices $P > 0$, $Q_i > 0$, $R_i > 0$ $(i = 1, 2)$, a diagonal matrix $S > 0$, and constants $p > 0$ and $c > 0$ such that

$$\begin{pmatrix} \bar{\Omega}_{11}^1 & I_N \otimes R_1 & \bar{\Omega}_{13}^1 & 0 & \bar{\Omega}_{15}^1 \\ * & \bar{\Omega}_{22}^1 & I_N \otimes R_2 & 0 & 0 \\ * & * & \bar{\Omega}_{33}^1 & I_N \otimes R_2 & \bar{\Omega}_{35}^1 \\ * & * & * & \bar{\Omega}_{44}^1 & 0 \\ * & * & * & * & \bar{\Omega}_{55}^1 \end{pmatrix} < 0 \quad (13)$$

where

$$\begin{aligned} \bar{\Omega}_{11}^1 &= I_N \otimes \Big(PA + A^{\mathrm{T}}P + Q_1 + HSH - R_1 \\ &\qquad + \tau^2 A^{\mathrm{T}}R_1A + p^2A^{\mathrm{T}}R_2A\Big) \\ \bar{\Omega}_{13}^1 &= -c(L+K)\otimes\Big(P + \tau^2A^{\mathrm{T}}R_1 + p^2A^{\mathrm{T}}R_2\Big) \\ \bar{\Omega}_{15}^1 &= I_N \otimes \Big(PB + \tau^2A^{\mathrm{T}}R_1B + p^2A^{\mathrm{T}}R_2B\Big) \\ \bar{\Omega}_{22}^1 &= I_N \otimes (Q_2 - Q_1 - R_1 - R_2) \\ \bar{\Omega}_{33}^1 &= -2I_N \otimes R_2 + c^2(L+K)^{\mathrm{T}}(L+K)\otimes\Big(\tau^2R_1 + p^2R_2\Big) \\ \bar{\Omega}_{35}^1 &= -c(L+K)^{\mathrm{T}}\otimes\Big(\tau^2R_1 + p^2R_2\Big)B \\ \bar{\Omega}_{44}^1 &= I_N \otimes (-Q_2 - R_2) \\ \bar{\Omega}_{55}^1 &= I_N \otimes \Big(-S + \tau^2B^{\mathrm{T}}R_1B + p^2B^{\mathrm{T}}R_2B\Big). \end{aligned}$$

In the case that each agent can receive quick responses from their neighbors and the leader. We assume that $\tau_k = 0$. Then, $0 \leq \tau_1(t) < p_1$ and $p_1 \leq \tau_2(t) < p_2$. Theorem 1 implies the following result.

Corollary 2: Under Assumptions 1 and 2, the error system (9) is exponentially mean-square stable if there exist matrices $P > 0$, $Q_i > 0$, and $R_i > 0$ $(i = 2, 3)$, a diagonal matrix $S > 0$, and constants $p_1 > 0$, $p_2 > 0$, $c > 0$, and $\beta \geq 0$ such that

$$\begin{pmatrix} \hat{\Omega}_{11}^1 & \hat{\Omega}_{12}^1 & 0 & \hat{\Omega}_{14}^1 & 0 & \hat{\Omega}_{16}^1 \\ * & \hat{\Omega}_{22}^1 & I_N \otimes R_1 & 0 & 0 & \hat{\Omega}_{26}^1 \\ * & * & \hat{\Omega}_{33}^1 & I_N \otimes R_2 & 0 & 0 \\ * & * & * & \hat{\Omega}_{44}^1 & I_N \otimes R_2 & \hat{\Omega}_{46}^1 \\ * & * & * & * & \hat{\Omega}_{55}^1 & 0 \\ * & * & * & * & * & \hat{\Omega}_{66}^1 \end{pmatrix} < 0$$

where

$$\begin{aligned} \hat{\Omega}_{11}^1 &= I_N \otimes \big(PA + A^{\mathrm{T}}P + Q_2 + HSH - R_2 + A^{\mathrm{T}}\digamma A\big) \\ \hat{\Omega}_{12}^1 &= -c\alpha(L+K)\otimes\big(P + A^{\mathrm{T}}\digamma\big) + I_N \otimes R_2 \\ \hat{\Omega}_{14}^1 &= -c(1-\alpha)(L+K)\otimes\big(P + A^{\mathrm{T}}\digamma\big) \\ \hat{\Omega}_{16}^1 &= I_N \otimes \big(PB + A^{\mathrm{T}}\digamma B\big) \\ \hat{\Omega}_{22}^1 &= -2I_N \otimes R_2 + c^2\alpha(L+K)^{\mathrm{T}}(L+K)\otimes\digamma \\ &\quad \hat{\Omega}_{26}^1 - c\alpha(L+K)^{\mathrm{T}}\otimes\digamma B \\ \hat{\Omega}_{33}^1 &= I_N \otimes (Q_3 - Q_2 - R_2 - R_1) \\ \hat{\Omega}_{44}^1 &= -2I_N \otimes R_3 + c^2(1-\alpha)(L+K)^{\mathrm{T}}(L+K)\otimes\digamma \\ \hat{\Omega}_{46}^1 &= -c(1-\alpha)(L+K)^{\mathrm{T}}\otimes\digamma B \\ \hat{\Omega}_{55}^1 &= I_N \otimes (-Q_3 - R_3) \\ \hat{\Omega}_{66}^1 &= I_N \otimes \big(-S + B^{\mathrm{T}}\digamma B\big); \quad \digamma = p_1^2R_2 + (p_2 - p_1)^2R_3. \end{aligned}$$

Proof: Choose the Lyapunov–Krasovskii functional candidate (10) by setting $\tau = 0$. Then, by Theorem 1, the conclusion of Corollary 2 follows. This completes the proof. ■

If the transmittal delay can be neglected in a network of multiagent systems with deterministic sampling, we have the following corollary.

Corollary 3: Under Assumptions 1 and 2, the error system (12) is exponentially stable if there exist matrices $P > 0$, $Q_2 > 0$, $R_2 > 0$, a diagonal matrix $S > 0$, and constants $p > 0$ and $c > 0$ such that

$$\begin{pmatrix} \tilde{\Omega}_{11}^1 & \tilde{\Omega}_{12}^1 & 0 & \tilde{\Omega}_{14}^1 \\ * & \tilde{\Omega}_{22}^1 & I_N \otimes R_2 & \tilde{\Omega}_{24}^1 \\ * & * & -I_N \otimes (Q_2 + R_2) & \tilde{\Omega}_{44}^1 \end{pmatrix} < 0 \tag{14}$$

where

$$\begin{aligned} \tilde{\Omega}_{11}^1 &= I_N \otimes \left(PA + A^{\mathrm{T}}P + Q_2 + HSH - R_2 + p^2 A^{\mathrm{T}} R_2 A\right) \\ \tilde{\Omega}_{12}^1 &= -c(L+K) \otimes \left(P + p^2 A^{\mathrm{T}} R_2\right) + I_N \otimes R_2 \\ \tilde{\Omega}_{14}^1 &= I_N \otimes \left(PB + p^2 A^{\mathrm{T}} R_2 B\right) \\ \tilde{\Omega}_{22}^1 &= -2 I_N \otimes R_2 + c^2 p^2 (L+K)^{\mathrm{T}} (L+K) \otimes R_2 \\ \tilde{\Omega}_{24}^1 &= -cp^2 (L+K)^{\mathrm{T}} \otimes R_2 B \\ \tilde{\Omega}_{44}^1 &= I_N \otimes \left(-S + p^2 B^{\mathrm{T}} R_2 B\right). \end{aligned}$$

Remark 3: Compared Theorem 1 with Corollary 2, and Corollary 1 with Corollary 4, results with delays are complex. Usually, the time delay has a negative effect on stability. In the case of stochastic sampling, it means smaller sampling intervals are required, which is verified in Example 1.

Note that the dimension of the matrix Ω_1 in Theorem 1 is $7Nn \times 7Nn$. When the network is large, it is not easy to be verified. Some low-dimensional conditions are expected. Next, we will show how to deal with the network configuration matrix with the purpose of reducing the order of Ω_1 and revealing the relationship between the sampling scheme and the network parameters including a network topology L, a pinning matrix K, and a coupling strength c.

B. Consensus Under Undirected Graph

Theorem 2: Under Assumptions 1 and 2, the error system (9) under an undirected graph is exponentially mean square stable if there exist matrices $P > 0$, $Q_j > 0$, $R_j > 0$, $(j = 1, 2, 3)$, a diagonal matrix $S > 0$, and constants $p_1 > 0$, $p_2 > 0$, $c > 0$, and $\beta \geq 0$ such that

$$\Omega_2(i) < 0 \tag{15}$$

where $\Omega_2(i)$ is described by

$$\begin{pmatrix} \Omega_{11}^2 & R_1 & \Omega_{13}^2(i) & 0 & \Omega_{15}^2(i) & 0 & \Omega_{17}^2 \\ * & \Omega_{22}^2 & R_2 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33}^2(i) & R_2 & 0 & 0 & \Omega_{37}^2(i) \\ * & * & * & \Omega_{44}^2 & R_3 & 0 & 0 \\ * & * & * & * & \Omega_{55}^2(i) & R_3 & \Omega_{57}^2(i) \\ * & * & * & * & * & \Omega_{66}^2 & 0 \\ * & * & * & * & * & * & \Omega_{77}^2 \end{pmatrix}$$

$$\begin{aligned} \Omega_{11}^2 &= PA + A^{\mathrm{T}}P + Q_1 + HSH - R_1 + A^{\mathrm{T}} \digamma A \\ \Omega_{17}^2 &= PB + A^{\mathrm{T}} \digamma B \\ \Omega_{22}^2 &= Q_2 - Q_1 - R_1 - R_2 \\ \Omega_{44}^2 &= Q_3 - Q_2 - R_2 - R_3 \\ \Omega_{66}^2 &= -Q_3 - R_3 \\ \Omega_{77}^2 &= -S + B^{\mathrm{T}} \digamma B \\ \Omega_{13}^2(i) &= -c\alpha\lambda_i \left(P + A^{\mathrm{T}} \digamma\right) \\ \Omega_{15}^2(i) &= -c(1-\alpha)\lambda_i \otimes \left(P + A^{\mathrm{T}} \digamma\right) \\ \Omega_{33}^2(i) &= -2R_2 + c^2 \alpha \lambda_i^2 \digamma \\ \Omega_{37}^2(i) &= -c\alpha\lambda_i \digamma B \\ \Omega_{55}^2(i) &= -2R_3 + c^2 (1-\alpha) \lambda_i^2 \digamma \\ \Omega_{57}^2(i) &= -c(1-\alpha)\lambda_i \digamma B \\ \digamma &= \tau^2 R_1 + p_1^2 R_2 + (p_2 - p_1)^2 R_3 \end{aligned}$$

where λ_i $(i = 1, 2, \ldots, N)$ are the eigenvalues of $L + K$ and $0 < \lambda_1 \leq \cdots \leq \lambda_N$.

Proof: Based on [9], according to Assumption 2 all eigenvalues of the matrix $L + K$ are positive with $0 < \lambda_1 \leq \cdots \leq \lambda_N$. Then, there exists an orthogonal matrix $\Phi = (\phi_1, \phi_2, \ldots, \phi_N) \in R^{N \times N}$, such that $\Phi^{\mathrm{T}}(L + K)\Phi = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ and $\Phi^{\mathrm{T}}(L + K)(L + K)\Phi = \mathrm{diag}\{\lambda_1^2, \lambda_2^2, \ldots, \lambda_N^2\}$.

Pre- and post-multiply both sides of Ω_1 with $\hat{\Phi} = \mathrm{diag}\{\Phi^{\mathrm{T}} \otimes I_n, \Phi^{\mathrm{T}} \otimes I_n, \Phi^{\mathrm{T}} \otimes I_n, \Phi^{\mathrm{T}} \otimes I_n, \Phi^{\mathrm{T}} \otimes I_n, \Phi^{\mathrm{T}} \otimes I_n\}$ and its transpose, respectively. Note that $\hat{\Phi}\Omega_1\hat{\Phi}^{\mathrm{T}} < 0$ is equivalent to $\Omega_2(i) < 0$, $i = 1, 2, \ldots, N$. This completes the proof. ■

By taking the advantage of the network topology, Theorem 2 provides a low-dimensional condition, which is simpler and easier to be verified. Moreover, effects of network topology L, pinning matrix K, and coupling strength c on consensus are shown. Some deep insights are addressed as follows.

Remark 4: By Theorem 2, the following sampled-data leader-following consensus problem:

$$\begin{aligned} L &: \dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)) \\ F &: \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) - c\sum_{j=1}^{N} l_{ij}\big(x_j(t_k) - x_i(t_k)\big) \\ &\qquad - ck_i(x_i(t_k) - x_0(t_k)), \ \forall\, t \in [t_k + \tau, t_{k+1} + \tau) \end{aligned} \tag{16}$$

with a leader agent L and follower agents F can be recast into N sampled-data master–slave synchronization of the following systems:

$$\begin{aligned} \mathcal{M} &: \dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)) \\ \mathcal{S} &: \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) \\ &\qquad - c\lambda_i(x_i(t_k) - x_0(t_k)), \ \forall\, t \in [t_k + \tau, t_{k+1} + \tau) \end{aligned} \tag{17}$$

with the master system $\mathcal{M}$ and the slave system $\mathcal{S}$. It is obvious that systems in (17) are decoupled from the original system (16). The eigenvalues λ_i $(i = 1, 2, \ldots, N)$ of $L + K$ identify the network structure and the pinning strategy. The decoupled method results in reduced-order matrices in Theorem 2, which are simple and easily applicable. Moreover, the influence of the network structure and the pinning strategy is shown via its eigenvalues. The effect of the coupling strength is also presented.

Remark 5: It is pivotal to select proper sampling periods and their occurrence probability to stabilize the error system (9), as shown in [30]. According to Theorem 1, such sampling periods p_1 and p_2 and the occurrence probability β can be obtained with a given network topology by solving (11). However, the dimension of Ω_1 is high and depends on the number of agents. The difficulty of its solvability increases as the number of agents increases. With the result of Theorem 2, a dimension-reduced condition is provided with the dimension $7n \times 7n$. Although it includes N matrix inequalities, we are able to solve the problem efficiently. Specially, for a given network topology L and a pinning matrix K. Assume $c\lambda_i(L+K) \in [m_1, m_2],\ i = 1, 2, \ldots, N$. One can solve (15) with MATLAB linear matrix inequality (LMI) Toolbox by replacing $c\lambda_i = m_1$ or $c\lambda_i = m_2$. Then a possible set of $p_1,\ p_2$, and β can be fixed. In this sense, the sampled-data leader-following consensus problem (16) can be recast into the following master–slave synchronization problem:

$$\begin{aligned} \mathcal{M} &: \dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)) \\ \mathcal{S} &: \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) \\ &\qquad - m_i(x_i(t_k) - x_0(t_k)),\ \forall\ t \in [t_k + \tau, t_{k+1} + \tau) \end{aligned} \tag{18}$$

where $i = 1, 2$. It is shown that to stabilize the network of agents, one can effectively choose $p_1,\ p_2$, and β to stabilize the system (18) with only two independent followers involved.

IV. Network Design

Theorems 1 and 2 emphasize the active role of the sampling scheme. However, the contribution of network topology and pinning strategy to consensus is unclear. We are now in a position to discuss the positive effect of network topology and pinning strategy on consensus.

Theorem 3: Under Assumptions 1 and 2, for given $p_1,\ p_2$, and β if there exist constants $m_2 \geq m_1 > 0$, for any $m \in [m_1, m_2]$, there exist matrices $P > 0,\ Q_i > 0,\ R_i > 0,\ (i = 1, 2, 3)$, and a diagonal matrix $S > 0$ such that

$$\Omega_3 = \begin{pmatrix} \Omega_{11}^3 & R_1 & \Omega_{13}^3 & 0 & \Omega_{15}^3 & 0 & \Omega_{17}^3 \\ * & \Omega_{22}^3 & R_2 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33}^3 & R_2 & 0 & 0 & \Omega_{37}^3 \\ * & * & * & \Omega_{44}^3 & R_3 & 0 & 0 \\ * & * & * & * & \Omega_{55}^3 & R_3 & \Omega_{57}^3 \\ * & * & * & * & * & \Omega_{66}^3 & 0 \\ * & * & * & * & * & * & \Omega_{77}^3 \end{pmatrix} < 0$$

$$\begin{aligned} \Omega_{11}^3 &= PA + A^{\mathrm{T}}P + Q_1 + HSH - R_1 + A^{\mathrm{T}}\digamma A \\ \Omega_{13}^3 &= -\alpha m\big(P + A^{\mathrm{T}}\digamma\big) \\ \Omega_{15}^3 &= -(1-\alpha)m \otimes \big(P + A^{\mathrm{T}}\digamma\big) \\ \Omega_{17}^3 &= PB + A^{\mathrm{T}}\digamma B \\ \Omega_{22}^3 &= Q_2 - Q_1 - R_1 - R_2 \\ \Omega_{33}^3 &= -2R_2 + \alpha m^2 \digamma \\ \Omega_{37}^3 &= -\alpha m \digamma B \\ \Omega_{44}^3 &= Q_3 - Q_2 - R_2 - R_3 \\ \Omega_{55}^3 &= -2R_3 + (1-\alpha)m^2 \digamma \\ \Omega_{57}^3 &= -(1-\alpha)m \digamma B \\ \Omega_{66}^3 &= -Q_3 - R_3 \\ \Omega_{77}^3 &= -S + B^{\mathrm{T}}\digamma B \\ \digamma &= \tau^2 R_1 + p_1^2 R_2 + (p_2 - p_1)^2 R_3. \end{aligned}$$

Then, the error system (9) under an undirected graph achieves exponential mean-square stability if the pinning matrix K and the coupling strength c satisfy

$$\frac{\lambda_{\min}(L+K)}{\lambda_{\max}(L+K)} \geq \frac{m_1}{m_2} \tag{19}$$

and

$$\frac{m_1}{\lambda_{\min}(L+K)} \leq c \leq \frac{m_2}{\lambda_{\max}(L+K)}. \tag{20}$$

Proof: From (19), there exist coupling strengths c satisfying (20). Since $c\lambda_{\min}(L+K) \leq c\lambda_i \leq c\lambda_{\max}(L+K)$ (λ_i is defined in Theorem 2), according to (20), it is easy to obtain that

$$m_1 \leq c\lambda_i \leq m_2.$$

Therefore, for any $c\lambda_i,\ i = 1, 2, \ldots, N$, $\Omega_3 < 0$ holds. Then by Theorem 2, the conclusion of Theorem 3 follows. This completes the proof. ■

Remark 6: By [33, Lemma 4] and with the fact that $(1-\rho)m_1^2 + \rho m_2^2 \geq [(1-\rho)m_1 + \rho m_2]^2,\ \forall\ \rho \in [0, 1]$. $\Omega_3 < 0$ holds for $m = m_1$ and $m = m_2$ is sufficient to ensure $\Omega_3 < 0$ holds for $m \in [m_1, m_2]$. It is easy to get the lower bound m_1 and the upper bound m_2 of m by appropriate algorithms.

Remark 7: Theorem 3 deals with the problem of how to design network parameters to stabilize the multiagent systems for a given sampling scheme. Indeed, based on Theorem 3, the sampled-data leader-following consensus problem (16) is equivalent to the following master–slave synchronization problem:

$$\begin{aligned} \mathcal{M} &: \dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)) \\ \mathcal{S} &: \dot{x}_1(t) = Ax_1(t) + Bf(x_1(t)) \\ &\qquad - m(x_1(t_k) - x_0(t_k)),\ \forall\ t \in [t_k + \tau, t_{k+1} + \tau). \end{aligned} \tag{21}$$

The feedback gain m is to be determined. The allowable range of m is needed for the network design. It is shown that even if a system cannot be stabilized under a given p_1, p_2, and β, one can effectively design the communication network to facilitate the synchronous process.

In the case of $\tau = 0$, we have the following corollary.

Corollary 4: Under Assumptions 1 and 2, for given $p_1,\ p_2$, and β if there exist constants $m_2 \geq m_1 > 0$, for any $m \in [m_1, m_2]$, there exist matrices $P > 0,\ Q_i > 0,\ R_i > 0,\ (i = 2, 3)$, and a diagonal matrix $S > 0$ such that

$$\hat{\Omega}_3 = \begin{pmatrix} \hat{\Omega}_{11}^3 & \hat{\Omega}_{12}^3 & 0 & \hat{\Omega}_{14}^3 & 0 & \hat{\Omega}_{16}^3 \\ * & \hat{\Omega}_{22}^3 & R_2 & 0 & 0 & \hat{\Omega}_{26}^3 \\ * & * & \hat{\Omega}_{33}^3 & R_3 & 0 & 0 \\ * & * & * & \hat{\Omega}_{44}^3 & R_3 & \hat{\Omega}_{46}^3 \\ * & * & * & * & -Q_3 - R_3 & 0 \\ * & * & * & * & * & \hat{\Omega}_{66}^3 \end{pmatrix} < 0 \tag{22}$$

where

$$\begin{aligned}
\hat{\Omega}_{11}^3 &= PA + A^{\mathrm{T}}P + Q_2 + HSH - R_2 + A^{\mathrm{T}}\digamma A\\
\hat{\Omega}_{12}^3 &= -\alpha m\big(P + A^{\mathrm{T}}\digamma\big) + R_2\\
\hat{\Omega}_{14}^3 &= -(1-\alpha)m\big(P + A^{\mathrm{T}}\digamma\big)\\
\hat{\Omega}_{16}^3 &= PB + A^{\mathrm{T}}\digamma B\\
\hat{\Omega}_{22}^3 &= -2R_2 + \alpha m^2\digamma\\
\hat{\Omega}_{26}^3 &= -\alpha m\digamma B\\
\hat{\Omega}_{33}^3 &= Q_3 - Q_2 - R_2 - R_3\\
\hat{\Omega}_{44}^3 &= -2R_3 + (1-\alpha)m^2\digamma\\
\hat{\Omega}_{46}^3 &= -(1-\alpha)m\digamma B\\
\hat{\Omega}_{66}^3 &= -S + B^{\mathrm{T}}\digamma B\\
\digamma &= p_1^2R_2 + (p_2-p_1)^2R_3.
\end{aligned}$$

Then, the error system (9) under an undirected graph achieves exponential mean-square stability if the pinning matrix K and the coupling strength c satisfy (19) and (20).

Remark 8: It is worth mentioning that the dimensions of the LMIs in Theorem 3 and Corollary 4 are independent of the scale of the network. In fact, master–salve synchronization (21) is ensured if $\Omega_3 < 0$ holds. In this sense, we have successfully reduce the problem of leader-following consensus with one leader and N followers into master–slave synchronization between a master system and a slave one, which strongly facilitates the solvability for large-scale networked agents. Moreover, the guidance of how to choose the controlled agents and to design the coupling strength is provided, enhancing potential applications. Its effectiveness will be illustrated by Example 2.

V. Numerical Simulations

In order to show the effectiveness of the derived results, we consider the following cellular neural network as the leader:

$$\dot{x}_0(t) = Ax_0(t) + Bf(t, x_0(t)) \tag{23}$$

where $x_0(t) = (x_{01}(t), x_{02}(t), x_{03}(t))^{\mathrm{T}}$ and $f(x_0(t)) = (q(x_{01}(t)), q(x_{02}(t)), q(x_{03}(t)))^{\mathrm{T}}$ with $q(x_{0i}(t)) = (1/2)(|x_{0i}(t)+1| - |x_{0i}(t)-1|)$, $i = 1, 2, 3$

$$A = \begin{pmatrix} -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1.25 & -3.2 & -3.2\\ -3.2 & 1.1 & -4.4\\ -3.2 & 4.4 & 1.0 \end{pmatrix}.$$

Fig. 2 shows the trajectory of the leader with initial values $x_{01}(0) = 0.1$, $x_{02}(0) = 0.2$, and $x_{03}(0) = -0.1$.

Example 1: Application to multiagent systems with an asymmetric network.

Consider a group of multiagent systems consisting four followers. The Laplacian matrix is

$$L = \begin{pmatrix} 1 & 0 & -1 & 0\\ -1 & 1 & 0 & 0\\ -1 & -1 & 2 & 0\\ -1 & 0 & -1 & 2 \end{pmatrix}.$$

The dynamics of every follower satisfies (23) but with different initial conditions.

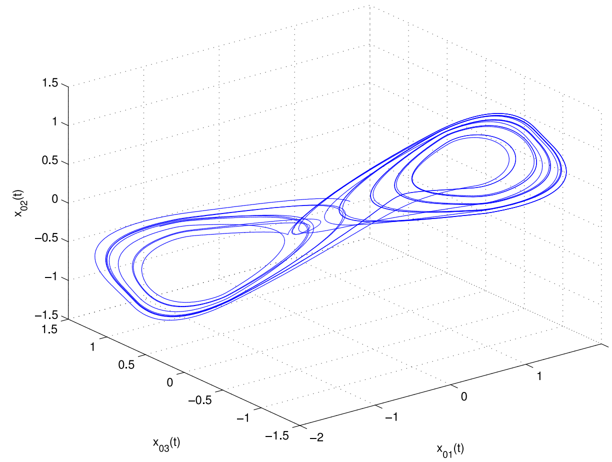

Fig. 2. Trajectory of the leader.

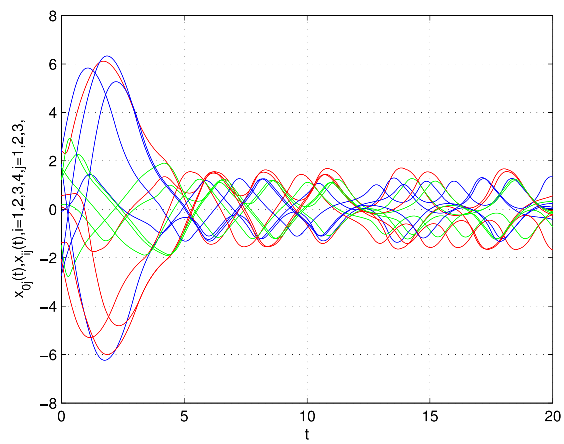

Fig. 3. Time evolutions of state variables of four followers and the leader without control.

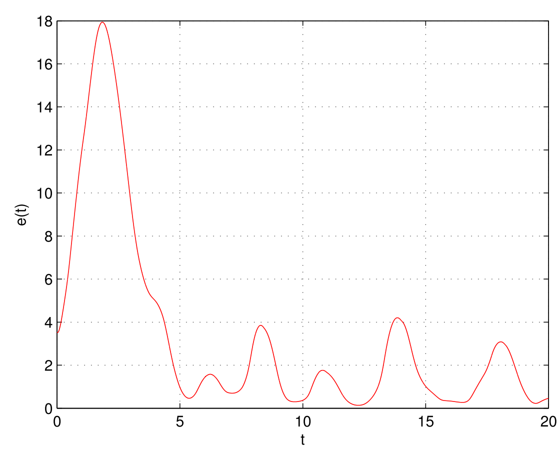

Fig. 4. Time evolution of $e(t)$ without control.

The average error between the followers and the leader is defined by

$$e(t) = \frac{1}{4\times 3}\sum_{i=1}^{4}\sum_{j=1}^{3}\big(x_{ij}(t) - x_{0j}(t)\big)^2. \tag{24}$$

Figs. 3 and 4 show that the leader-follower consensus cannot be achieved without the controller. In order to reach an

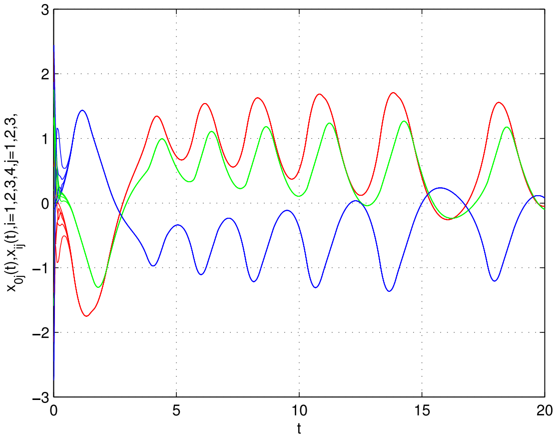

Fig. 5. Time evolutions of state variables of four followers and the leader with control in Example 1.

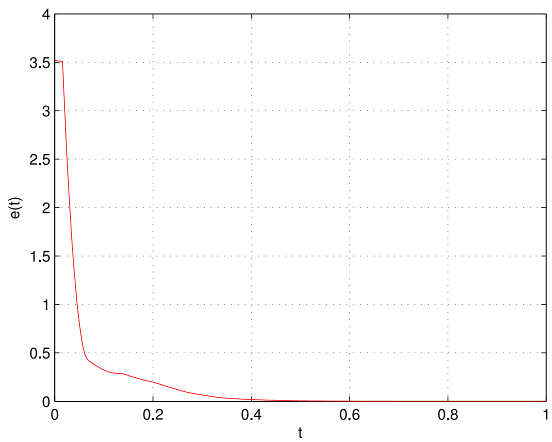

Fig. 6. Time evolution of $e(t)$ with control in Example 1.

agreement, sampled-data control as in (3) is adopted. Set $c = 5,\ k_1 = 3,\ k_2 = 2,\ k_3 = 0$, and $k_4 = 0$, only agents 1 and 2 are pinned. Choose $p_1 = 0.01,\ p_2 = 0.05$, and $\beta = 0.8$. According to Theorem 1, solving (11) using MATLAB LMI Toolbox yields $t\min = -1.2753 \times 10^{-6}$ and the maximal allowable delay is $\tau = 0.016$.

Thus, one can conclude that leader-follower consensus in the mean square is achieved. Fig. 5 gives the simulation result for the state variables and time evolution of the error signal $e(t)$ is depicted in Fig. 6.

We compare the stochastic sampling with the deterministic sampling. By Corollary 1, the maximal sampling period is 0.018. With stochastic sampling, the larger sampling period 0.05 is allowed. We also compare the results with and without the transmittal delay. Without the transmittal delay, the maximal value of p_2 is 0.098 according to Corollary 2, comparing with $p_2 = 0.05$ in the case of $\tau = 0.016$ in Theorem 1. If the deterministic sampling is considered, the maximal sampling period without the transmittal delay is 0.034 according to Corollary 3, while with the transmittal delay, the value is 0.018 by Corollary 1. It is shown that with stochastic sampling, the allowable sampling period has a larger value, compared with the deterministic sampling. The transmittal delay has a negative effect on the selection of the sampling period.

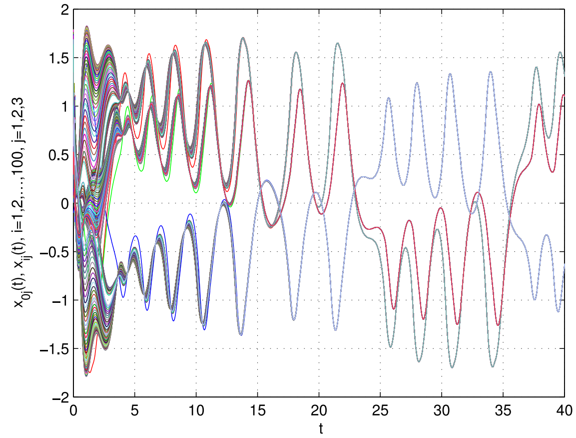

Fig. 7. Time evolutions of state variables of 100 followers and the leader with control in Example 2.

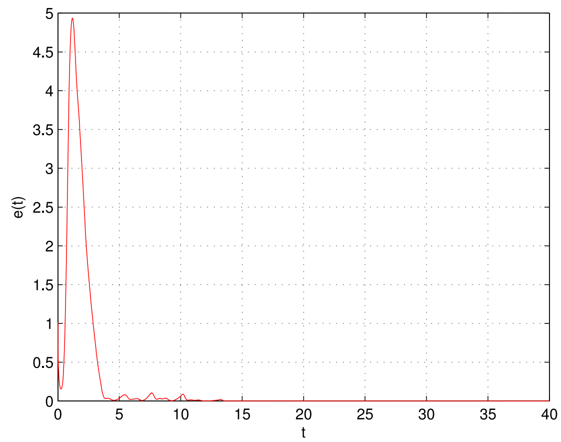

Fig. 8. Time evolution of $e(t)$ with control in Example 2.

Example 2: Application to multiagent systems with a symmetric network.

Now we choose a regular nearest neighbor network with 100 nodes, whose Laplacian matrix is described by

$$L = \begin{pmatrix} 4 & -1 & -1 & 0 & \cdots & 0 & -1 & -1 \\ -1 & 4 & -1 & -1 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & \cdots & 0 & -1 & -1 & 4 & -1 \\ -1 & -1 & 0 & \cdots & 0 & -1 & -1 & 4 \end{pmatrix}_{100\times 100}.$$

If Theorem 1 is applied, it will bring a big challenge to solve (11) as there are 100 agents.

Set $p_1 = 0.01,\ p_2 = 0.05,\ \beta = 0.8$, and $\tau = 0$. According to Corollary 4, it is easy to get the lower bound $m_1 = 5.553$ and the upper bound $m_2 = 57.792$. Then by (19), one has $(\lambda_{\min}(L+K)/\lambda_{\max}(L+K)) \geq (m_1/m_2) = 0.0961$. Choose 40% of the agents as the pinning nodes, such as nodes $(i-1)10+1, (i-1)10+4, (i-1)10+6, (i-1)10+9,\ i = 1, 2, \ldots, 9$, and set the corresponding pinning gains as $k_i = 10$. By simple calculation, one has that $\lambda_{\min}(L+K) = 1.4792$ and $\lambda_{\max}(L+K) = 15.3246$. The eigenratio is 0.0965. (19) is satisfied. Based on (19), one has that $3.7541 \leq c \leq 3.7712$. Set $c = 3.77$. We give the simulation results of state variables and the error signal $e(t)$ in Figs. 7 and 8, from which one can see that an agreement is reached from the leader with the 100 followers.

VI. Conclusion

The leader-following consensus problem of a class of nonlinear multiagent systems with probabilistic sampling is investigated. By utilizing an input-delay approach, the error system with sampled data is transformed into a continuous time-delay system with stochastic delays and its exponential mean-square stability is studied. A general consensus criterion is first derived for multiagent systems under a directed graph. Then, several special cases without transmittal delays or with the deterministic sampling are discussed. By taking the advantage of the symmetric network, a dimension-reduced condition is obtained for the case with undirected graphs. Furthermore, the network design problem is solved. The derived results confirm the positive role of the network in reaching consensus and provide an easy-verified criterion, facilitating its application in engineering. Possible application for these criteria in formation control of vertical taking-off and landing aircrafts will be explored.

Appendix

Proof of Theorem 1

The infinitesimal operator $\mathcal{L}$ of $V(t, e_t)$ is defined as

$$\mathcal{L}V(t, e_t) \triangleq \lim_{\Delta \to 0^+} \frac{\mathbb{E}\{V(t+\Delta, e_{t+\Delta})|e_t\} - V(t, e_t)}{\Delta}.$$

From (9) and (10), one has

$$\begin{aligned}
\mathcal{L}V(t, e(t)) = {} & 2e^{\mathrm{T}}(t)(I_N \otimes P)\varpi(t) + e^{\mathrm{T}}(t)(I_N \otimes Q_1)e(t) \\
& - e^{\mathrm{T}}(t-\tau)(I_N \otimes (Q_1 - Q_2))e(t-\tau) \\
& - e^{\mathrm{T}}(t-p_1-\tau)(I_N \otimes (Q_2 - Q_3))e(t-p_1-\tau) \\
& - e^{\mathrm{T}}(t-p_2-\tau)(I_N \otimes Q_3)e(t-p_2-\tau) \\
& + \varpi^{\mathrm{T}}(t)(I_N \otimes F)\varpi(t) + \rho^{\mathrm{T}}(t)(I_N \otimes F)\rho(t) \\
& - \tau \int_{t-\tau}^{t} \Big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_1)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_1)\rho(s)\Big)\mathrm{d}s \\
& - p_1 \int_{t-p_1-\tau}^{t-\tau} \Big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_2)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_2)\rho(s)\Big)\mathrm{d}s \\
& - (p_2 - p_1) \int_{t-p_2-\tau}^{t-p_1-\tau} \Big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_3)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_3)\rho(s)\Big)\mathrm{d}s.
\end{aligned} \tag{25}$$

By simple calculation, it is easy to obtain that

$$\begin{aligned}
& -\tau\mathbb{E}\bigg\{\int_{t-\tau}^{t}\Big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_1)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_1)\rho(s)\Big)ds\bigg\} \\
& = -\tau\mathbb{E}\bigg\{\int_{t-\tau}^{t} \dot{e}^{\mathrm{T}}(s)(I_N \otimes R_1)\dot{e}(s)\mathrm{d}s\bigg\} \\
& \le -\mathbb{E}\bigg\{\bigg(\int_{t-\tau}^{t} \dot{e}(s)ds\bigg)^{\mathrm{T}}(I_N \otimes R_1)\bigg(\int_{t-\tau}^{t} \dot{e}(s)ds\bigg)\bigg\} \\
& = \mathbb{E}\bigg\{\big(e^{\mathrm{T}}(t), e^{\mathrm{T}}(t-\tau)\big)\Psi_1\begin{pmatrix} e(t) \\ e(t-\tau)\end{pmatrix}\bigg\}
\end{aligned} \tag{26}$$

$$\begin{aligned}
& -p_1\mathbb{E}\bigg\{\int_{t-p_1-\tau}^{t-\tau} \varpi^{\mathrm{T}}(s)(I_N \otimes R_2)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_2)\rho(s)\mathrm{d}s\bigg\} \\
& = -p_1\mathbb{E}\bigg\{\int_{t-p_1-\tau}^{t-\tau} \dot{e}^{\mathrm{T}}(s)(I_N \otimes R_2)\dot{e}(s)\mathrm{d}s\bigg\}.
\end{aligned} \tag{27}$$

Using Jensen's inequality, one gets

$$\begin{aligned}
& -p_1\mathbb{E}\bigg\{\int_{t-p_1-\tau}^{t-\tau} \dot{e}^{\mathrm{T}}(s)(I_N \otimes R_2)\dot{e}(s)\mathrm{d}s\bigg\} \\
& \le \mathbb{E}\bigg\{-(t-\tau-(t-\tau_1(t))) \\
& \qquad \times \int_{t-\tau_1(t)}^{t-\tau} \dot{e}^{\mathrm{T}}(s)(I_N \otimes R_2)\dot{e}(s)\mathrm{d}s \\
& \qquad - ((t-\tau_1(t)) - (t-p_1-\tau)) \\
& \qquad \times \int_{t-p_1-\tau}^{t-\tau_1(t)} \dot{e}^{\mathrm{T}}(s)(I_N \otimes R_2)\dot{e}(s)\mathrm{d}s\bigg\} \\
& \le -\mathbb{E}\bigg\{\int_{t-\tau_1(t)}^{t-\tau} \dot{e}^{\mathrm{T}}(t)\mathrm{d}s\bigg\}(I_N \otimes R_2)\mathbb{E}\bigg\{\int_{t-\tau_1(t)}^{t-\tau} \dot{e}(s)\mathrm{d}s\bigg\} \\
& \quad - \mathbb{E}\bigg\{\int_{t-p_1-\tau}^{t-\tau_1(t)} \dot{e}^{\mathrm{T}}(t)\mathrm{d}s\bigg\}(I_N \otimes R_2)\mathbb{E}\bigg\{\int_{t-p_1-\tau}^{t-\tau_1(t)} \dot{e}(s)\mathrm{d}s\bigg\} \\
& = \mathbb{E}\bigg\{\big(e^{\mathrm{T}}(t-\tau), e^{\mathrm{T}}(t-\tau_1(t))\big)\Psi_2\begin{pmatrix} e(t-\tau) \\ e(t-\tau_1(t))\end{pmatrix}\bigg\} \\
& \quad + \mathbb{E}\bigg\{\big(e^{\mathrm{T}}(t-\tau_1(t)), e^{\mathrm{T}}(t-p_1-\tau)\big) \\
& \qquad \times \Psi_2\begin{pmatrix} e(t-\tau_1(t)) \\ e(t-p_1-\tau)\end{pmatrix}\bigg\}.
\end{aligned} \tag{28}$$

Similarly, one has

$$\begin{aligned}
& -(p_2-p_1)\ \mathbb{E}\bigg\{\int_{t-p_2-\tau}^{t-p_1-\tau}\Big(\varpi^{\mathrm{T}}(s)(I_N \otimes R_3)\varpi(s) \\
& \qquad + \rho^{\mathrm{T}}(s)(I_N \otimes R_3)\rho(s)\Big)\mathrm{d}s\bigg\} \\
& \le \mathbb{E}\bigg\{\big(e^{\mathrm{T}}(t-p_1-\tau), e^{\mathrm{T}}(t-\tau_2(t))\big)\Psi_3\begin{pmatrix} e(t-p_1-\tau) \\ e(t-\tau_2(t))\end{pmatrix}\bigg\} \\
& \quad + \mathbb{E}\bigg\{\big(e^{\mathrm{T}}(t-\tau_2(t)), e^{\mathrm{T}}(t-p_2-\tau)\big)\Psi_3\begin{pmatrix} e(t-\tau_2(t)) \\ e(t-p_2-\tau)\end{pmatrix}\bigg\}
\end{aligned} \tag{29}$$

where

$$\Psi_i = \begin{pmatrix} -I_N \otimes R_i & I_N \otimes R_i \\ I_N \otimes R_i & -I_N \otimes R_i \end{pmatrix}$$

$i = 1, 2, 3$. By Assumption 1, for a diagonal matrix $S > 0$, we have

$$e^{\mathrm{T}}(t)(I_N \otimes HSH)e(t) - g^{\mathrm{T}}(e(t))(I_N \otimes S)g(e(t)) \ge 0 \tag{30}$$

where $H = (h_{ij})_{n \times n}$.

Let $\xi(t) = (e^{\mathrm{T}}(t), e^{\mathrm{T}}(t-\tau), e^{\mathrm{T}}(t-\tau_1(t)), e^{\mathrm{T}}(t-p_1-\tau), e^{\mathrm{T}}(t-\tau_2(t)), e^{\mathrm{T}}(t-p_2-\tau)$, and $g^{\mathrm{T}}(e(t)))^{\mathrm{T}}$. Taking expectation on both sides of (25) and substituting (27)–(30) into (25) yield

$$\mathbb{E}\{\mathcal{L}V(t, e_t)\} \le \mathbb{E}\big\{\xi^{\mathrm{T}}(t)\Omega_1\xi(t)\big\}. \tag{31}$$

In view of (11), one has

$$\mathbb{E}\{\mathcal{L}V(t, e_t)\} \leq -\lambda \mathbb{E}\left\{\|e(t)\|^2\right\}$$

where $\lambda = -\lambda_{\min}(-\Omega_1)$. We supplement the states $x_j(\theta)$ on $[-2p_2-2\tau, -p_2-\tau]$ as $x_j(\theta) = x_j(0),\ -2p_2-2\tau \leq \theta \leq -p_2-\tau,\ j = 0, 1, 2, \ldots, N$. By Lemma 1 in [30], the exponential mean-square stability of the error system (9) is guaranteed. This completes the proof.

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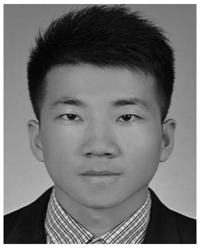
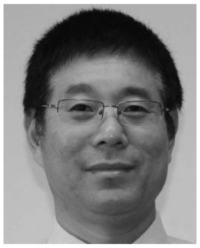
Wangli He (M'10) received the B.S. degree in information and computing science and the Ph.D. degree in applied mathematics from Southeast University, Nanjing, China, in 2005 and 2010, respectively.

From 2010 to 2011, she was a Post-Doctoral Research Fellow with the School of Information Science and Engineering, East China University of Science and Technology, Shanghai, China. From 2010 to 2011, she was a Visiting Post-Doctoral Research Fellow with the Centre for Intelligent and Networked Systems, Central Queensland University, North Rockhampton, QLD, Australia. From 2013 to 2014, she was a Research Associate with the Department of Mechanical Engineering, University of Hong Kong, Hong Kong. In 2014, she was a Research Fellow with the Centre for Chaos and Complex Networks, City University of Hong Kong, Hong Kong, for three months. In 2015, she was a Research Fellow with the Potsdam Institute for Climate Research Institute, Potsdam, Germany, for two months. She is currently an Associate Professor with the School of Information Science and Engineering, East China University of Science and Technology. Her current research interests include networked nonlinear systems, multiagent systems, cooperative control, impulsive control, and sampled-data control.



Biao Zhang was born in Hunan, China, in 1989. He received the B.S. degree in automation from the Hunan University of Arts and Science, Changde, China, in 2012. He is currently pursuing the M.S. degree in control science and engineering with the East China University of Science and Technology, Shanghai, China.

His current research interests include sampled-data control and multiagent systems.



Qing-Long Han (M'09–SM'13) received the B.Sc. degree in mathematics from Shandong Normal University, Jinan, China, in 1983, and the M.Sc. and Ph.D. degrees in control engineering and electrical engineering from the East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively.

From 1997 to 1998, he was a Post-Doctoral Researcher Fellow with LAII-ESIP (now LIAS-ENSIP), Université de Poitiers, Poitiers, France. From 1999 to 2001, he was a Research Assistant Professor with the Department of Mechanical and Industrial Engineering, Southern Illinois University Edwardsville, Edwardsville, IL, USA. From 2001 to 2014, he was a Laureate Professor, an Associate Dean of Research and Innovation with the Higher Education Division, and the Founding Director of the Centre for Intelligent and Networked Systems, Central Queensland University, North Rockhampton, QLD, Australia. In 2014, he joined Griffith University, Nathan, QLD, Australia, where he is currently the Deputy Dean of Research with the Griffith Sciences and a Professor with the Griffith School of Engineering. In 2010, he was appointed as the Chang Jiang (Yangtze River) Scholar Chair Professor by the Ministry of Education, Shanghai. In 2011, he was appointed as the 100 Talents Program Chair Professor by the Shanxi Province of China. He is one of the World's Most Influential Scientific Minds: 2014 and is a Highly Cited Researcher in the field of Engineering according to Thomson Reuters. His current research interests include networked control systems, neural networks, time-delay systems, multiagent systems, and complex systems.

Feng Qian received the B.Sc. degree in chemical automation and meters from the Nanjing Institute of Chemical Technology, Nanjing, China, in 1982, and the M.S. and Ph.D. degrees in automation from the East China Institute of Chemical Technology, Shanghai, China, in 1988 and 1995, respectively.

From 1999 to 2001, he was the Director of Automation Institute, East China University of Science and Technology, Shanghai, where he served as the Head of Scientific and Technical Department from 2001 to 2006. He is currently the Vice President of the East China University of Science and Technology, the Director of the Key Laboratory of Advanced Control and Optimization for Chemical Processes, Ministry of Education, Shanghai, and the Director of the Process System Engineering Research Centre, Ministry of Education. His current research interests include modeling, control, optimization and integration of petrochemical complex industrial process and the industrial applications, neural networks theory, real-time intelligent control technology and their applications on the ethylene, PTA, PET, and refining industries.

Prof. Qian is currently a member of China Instrument and Control Society, the Chinese Association of Higher Education, and China's PTA Industry Association. He is a fellow of the Chinese Academy of Engineering.

Jürgen Kurths studied mathematics from the University of Rostock, Rostock, Germany, and received the Ph.D. degree from the GDR Academy of Sciences, Berlin, Germany, in 1983.

He was a Full Professor with the University of Potsdam, Potsdam, Germany, from 1994 to 2008. Since 2008, he has been a Professor of Nonlinear Dynamics with Humboldt University, Berlin, and the Chair of Research Domain Transdisciplinary Concepts with the Potsdam Institute for Climate Impact Research, Potsdam. Since 2009, he has been the 6th Century Chair of Aberdeen University, Aberdeen, U.K. He has over 650 papers and an H-factor of 68. His current research interests include synchronization, complex networks, and time series analysis and their applications.

Dr. Kurths was a recipient of the Alexander von Humboldt Research Award from CSIR, India, in 2005, and the Honorary Doctorate from the Lobachevsky University of Nizhny Novgorod in 2008 and the State University of Saratov in 2012. He is an Editor of *PLoS ONE*, the *Journal of Nonlinear Science*, *CHAOS*, and the *European Physical Journal Special Topics*. He became a member of the Academia Europaea in 2010 and the Macedonian Academy of Sciences and Arts in 2012. He is a fellow of the American Physical Society.

Jinde Cao (M'07–SM'07–F'16) received the B.S. degree from Anhui Normal University, Wuhu, China, the M.S. degree from Yunnan University, Kunming, China, and the Ph.D. degree from Sichuan University, Chengdu, China, all in mathematics/applied mathematics, in 1986, 1989, and 1998, respectively.

From 1989 to 2000, he was with Yunnan University, where he was a Professor from 1996 to 2000. In 2000, he joined the Department of Mathematics, Southeast University, Nanjing, China. From 2001 to 2002, he was a Post-Doctoral Research Fellow with the Department of Automation and Computer-Aided Engineering, Chinese University of Hong Kong, Hong Kong. From 2006 to 2008, he was a Visiting Research Fellow and a Visiting Professor with the School of Information Systems, Computing and Mathematics, Brunel University, Uxbridge, U.K. In 2014, he was a Visiting Professor with the School of Electrical and Computer Engineering, RMIT University, Melbourne, VIC, Australia. He is currently a Distinguished Professor and a Doctoral Advisor with Southeast University. He has authored or co-authored over 400 journal papers and five edited books. His current research interests include nonlinear systems, neural networks, complex systems, complex networks, stability theory, and applied mathematics.

Dr. Cao was a recipient of the ISI Highly-Cited Researcher Award for mathematics, computer science, and engineering by Thomson Reuters. He was an Associate Editor of the IEEE TRANSACTIONS ON NEURAL NETWORKS, the *Journal of the Franklin Institute*, and *Neurocomputing*. He is an Associate Editor of the IEEE TRANSACTIONS ON CYBERNETICS, *Differential Equations and Dynamical Systems*, *Mathematics and Computers in Simulation*, and *Neural Networks*. He is a Reviewer of Mathematical Reviews and Zentralblatt-Math.